Spectrum Efficient Waveform Design with Application to Wireless Networks

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OUTLINE

- Introduction
- Nested Sampling
- Co-Prime Sampling
- Spectrum Efficiency
- Application to Wireless Networks
- Conclusions

Introduction

- The seminal work by Vaidyanathan and Pal on Co-Prime and Nested Samplers and Arrays has drawn lots of attentions in signal processing community.
- The Co-Prime and Nested Samplers and Arrays can be used to sample a wide-sense stationary signal sparsely, and then reconstruct the autocorrelation.
- Its applications are very limited so far, which only includes beamforming and direction finding and systems identification.
- We observed that it could be applied to spectrum efficient waveform design.

Nested Sampling

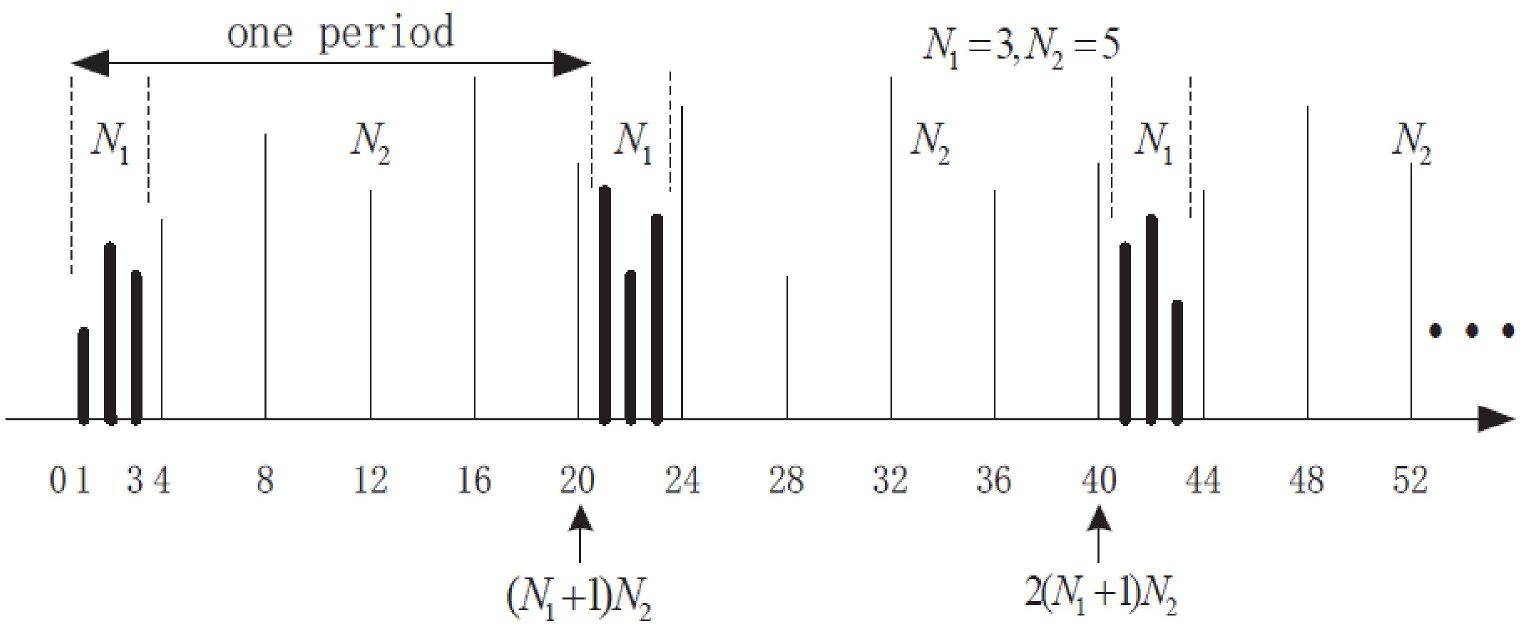


Fig. 1. Nested sampling with $N_1 = 3, N_2 = 5$.

- **an example of periodic sparse sampling using nested sampling structure**

Nested Sampling

- Non-uniform sampling
- Two levels of sampling density
- ✓ **the level 1 samples at the N_1 locations**
- ✓ **the level *2* samples at the N_2 locations**

$$1 \leq l \leq N_1, \text{for level 1}$$

$$(N_1 + 1)m, 1 \leq m \leq N_2, \text{for level 2}$$

Nested Sparse Sampling (Cont.)

- The cross-differences are given by

$$k = (N_1+1)m - l, 1 \le m \le N_2, 1 \le l \le N_1$$

- The cross-differences are in the following range

$$-[(N_1+1)N_2 - 1] \le k \le [(N_1+1)N_2 - 1]$$

- Except these integers and the corresponding negated versions (missing)

$$(N_1+1), 2(N_1+1), \cdots, (N2-1)(N_1+1)$$

Nested Sampling (Cont.)

- **the self differences among the second sequence could cover all of the missing differences**

$$(N_1+1)(m_1-m_2), 1 \le m_1, m_2 \le N_2$$

- **The differences (time lags) could be obtained from the cross-differences and the self-differences, which is a filled difference as**

$$-[(N_1+1)N_2-1] \le k \le [(N_1+1)N_2-1]$$

Nested Sampling (Cont.)

- In $(N_1+1)N_2T$ seconds, there are totally N_1+N_2 samples
- The average sampling rate (Sparse Sampling!)

$$f_{s,nested} = \frac{N_1+N_2}{(N_1+1)N_2T} \approx \frac{1}{N_1T} + \frac{1}{N_2T} < \frac{1}{T}$$

♦ **(the Nyquist sampling rate)**

$$T = 1/f_n$$

$$f_n \geq 2f_{max} \text{ is the Nyquist sampling frequency}$$

Co-Prime Sampling

- Two sub-Nyquist uniform samplers
- Two levels of sampling density

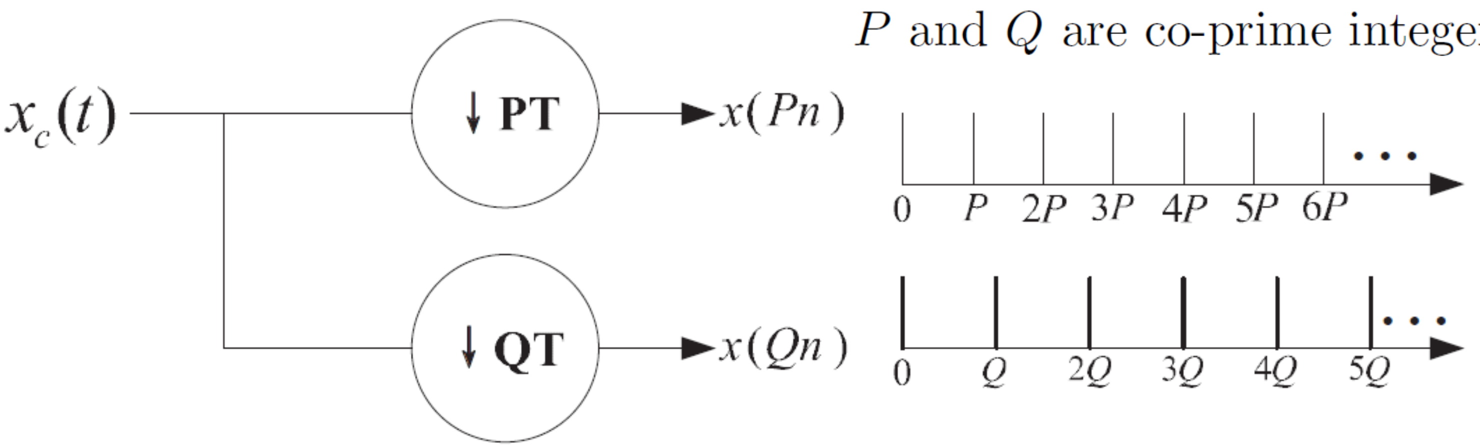


Fig. 2. Co-Prime sampling in the time domain.

Co-Prime Sampling (Cont.)

- **The differences of this pair:**

$$x(n_1, n_2) = Pn_1 - Qn_2, 0 \leq n_1 \leq Q-1, 0 \leq n_2 \leq P-1$$

Because of the co-primality of P and Q

$$-Q(P-1) \leq x(n_1, n_2) \leq P(Q-1)$$

- **For a wide sense stationary signal, its correlation can be calculated at all lags equal to the elements present in the difference.**

Co-Prime Sampling (Cont.)

Sampling spacings PT and QT respectively

Consider the product

$$x(Pn_1)x^*(Qn_2)$$

$x(Pn_1)$ and $x(Qn_2)$ comes from the first and the second sampler

$$k = Pn_1 - Qn_2$$

if n_1 and n_2 in the ranges $0 \leq n_1 \leq 2Q - 1$ and $0 \leq n_2 \leq P - 1$

k can achieve any interger value in the range

$$0 \leq k \leq PQ - 1$$

Co-Prime Sampling (Cont.)

- **For co-prime sampling, the two samplers collect *P* + *Q* samples in *PQT seconds***

- **The average sampling rate (Sparse Sampling!)**

$$f_{s,coprime} = \frac{P+Q}{PQT} = \frac{1}{PT} + \frac{1}{QT} < \frac{1}{T}$$

$$T = 1/f_n$$

$f_n \geq 2f_{max}$ is the Nyquist sampling frequency

PSD with Nested Sampling

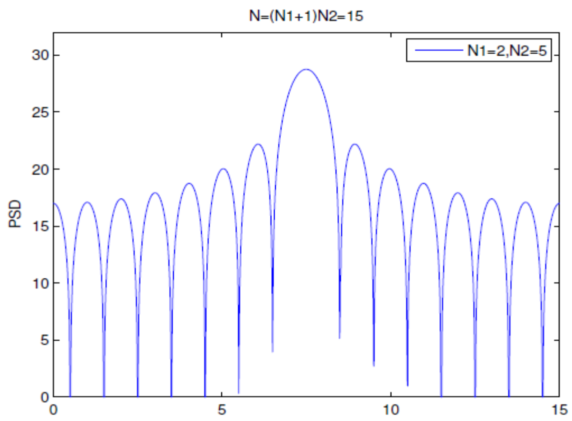


Fig. 5. PSD, N=15

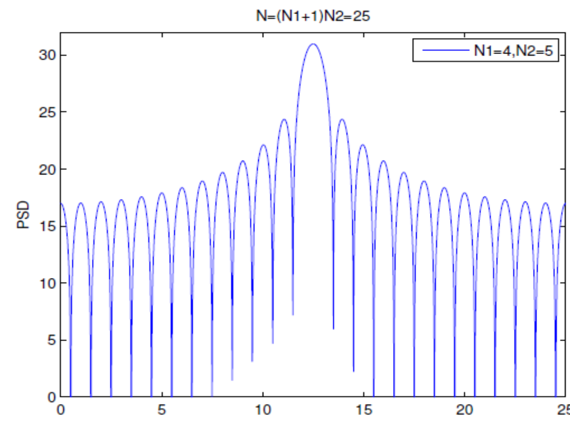


Fig. 6. PSD, N=25

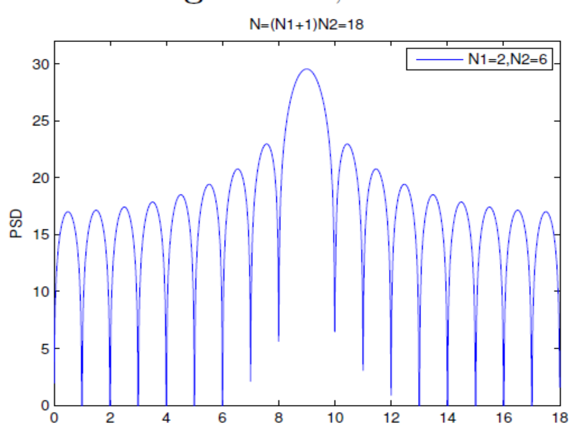


Fig. 7. PSD, N=18

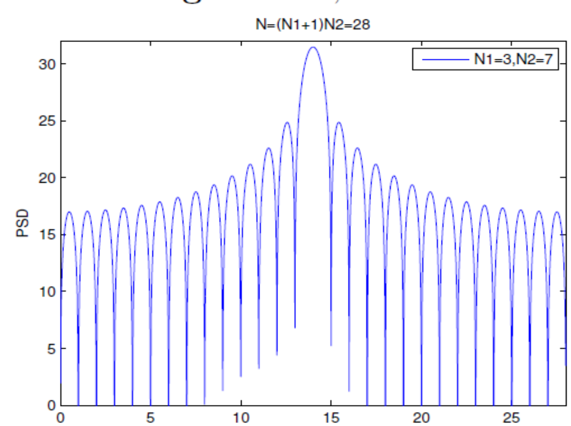


Fig. 8. PSD, N=28

Waveform Design

- **QPSK modulated signal with carrier frequency** $f_c = 400Hz$

$$s_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} cos[2\pi f_c t + (i-1)\frac{\pi}{2}]$$

where T_s is the symbol duration

- **The power spectral density of a QPSK signal using rectangular pulses can be expressed as**

$$P_{QPSK}(f) = \frac{E_s}{2}[(\frac{sin\pi(f-f_c)T_s}{\pi(f-f_c)T_s})^2 + (\frac{sin\pi(-f-f_c)T_s}{\pi(-f-f_c)T_s})^2]$$

PSD of QPSK Waveform

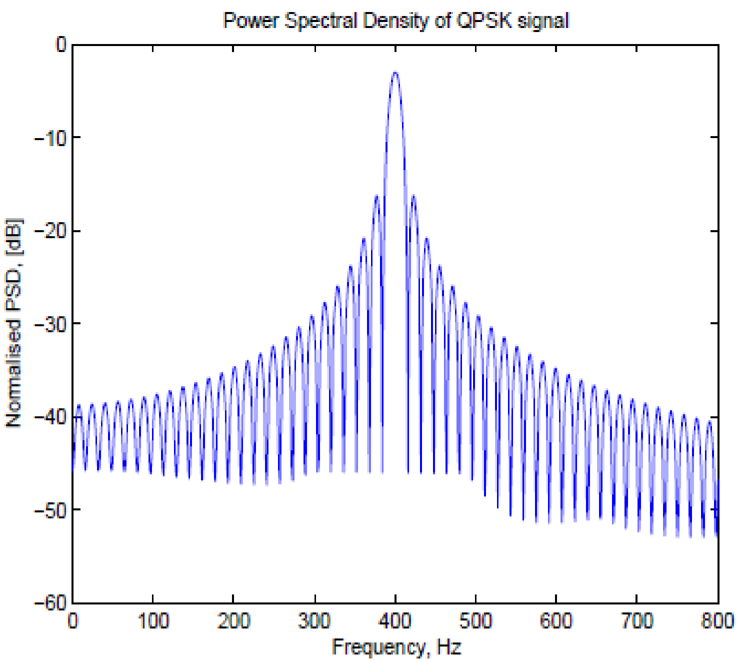


Fig. 9. PSD of the QPSK signal

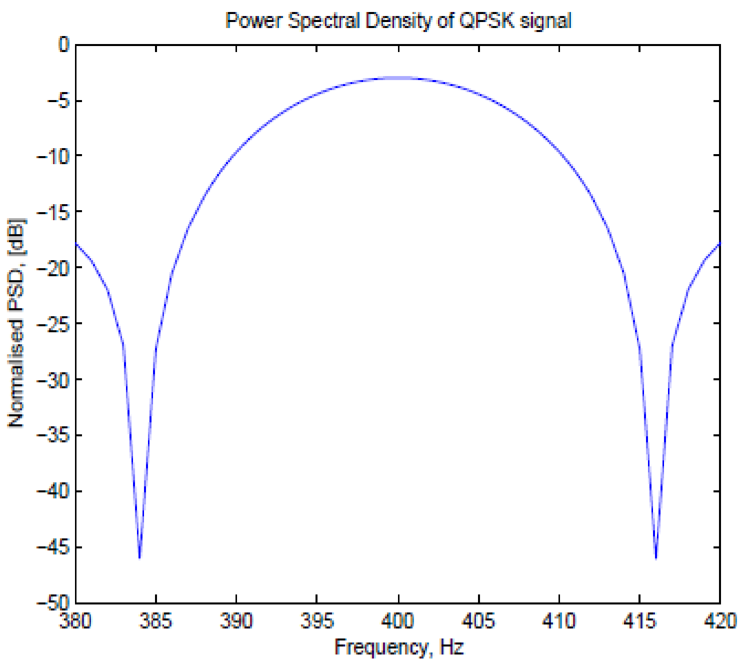


Fig. 10. Zoom in the main lobe of PSD for QPSK signal

- **BW:** $416 - 384 \approx 32Hz$

PSD of Nested Sampling QPSK Waveform

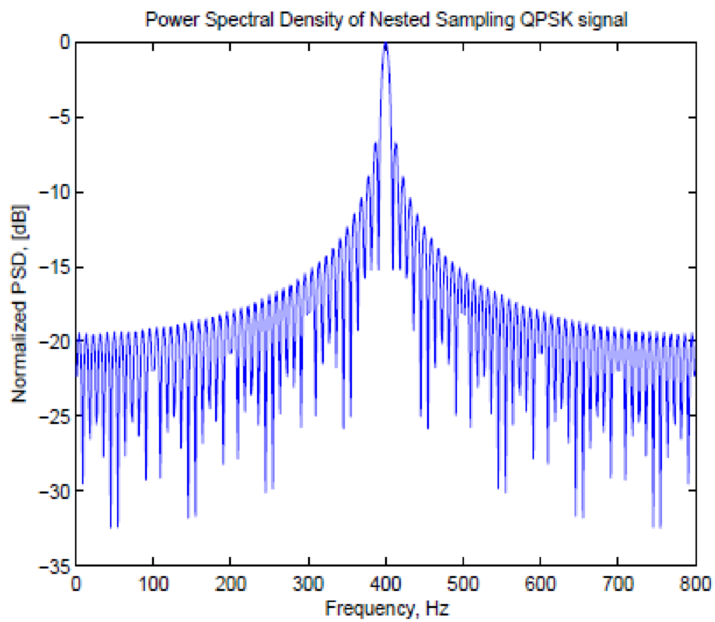


Fig. 13. PSD of Nested Sampling QPSK signal (N1=7,N2=11)

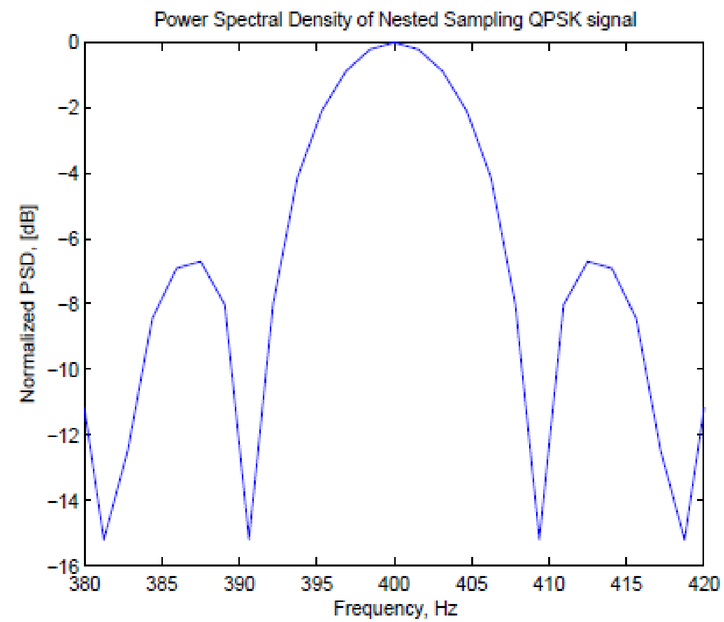


Fig. 14. Zoom in the main lobe of PSD for Nested Sampling QPSK signal(N1=7,N2=11)

- **BW:** $409 - 391 \approx 18Hz$

PSD of Co-Prime Sampling QPSK Waveform

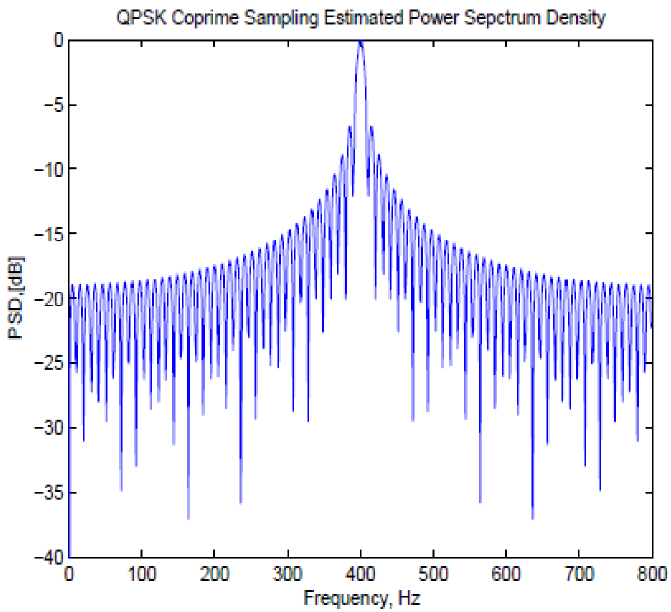


Fig. 15. PSD of Co-Prime Sampling QPSK signal (P=7,Q=11)

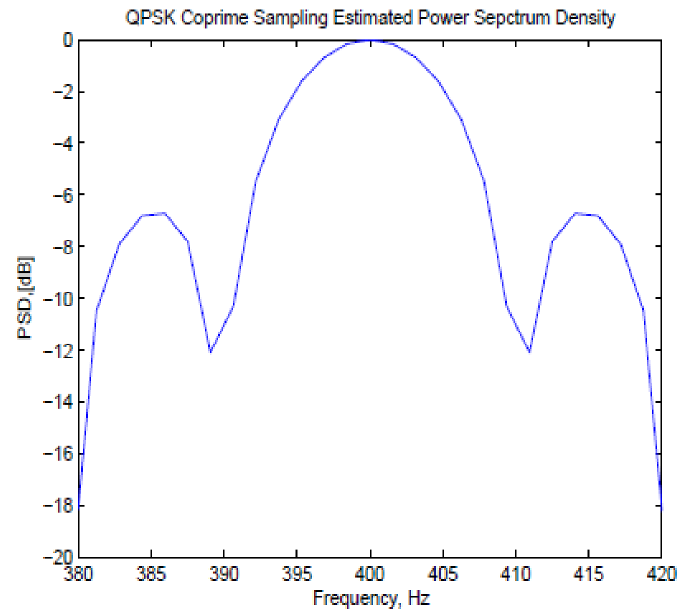


Fig. 16. Zoom in the main lobe of PSD for Co-Prime Sampling QPSK signal(P=7,Q=11)

- **BW: 411-389 ≈22Hz**

Nested Sampling PSD (normalized)

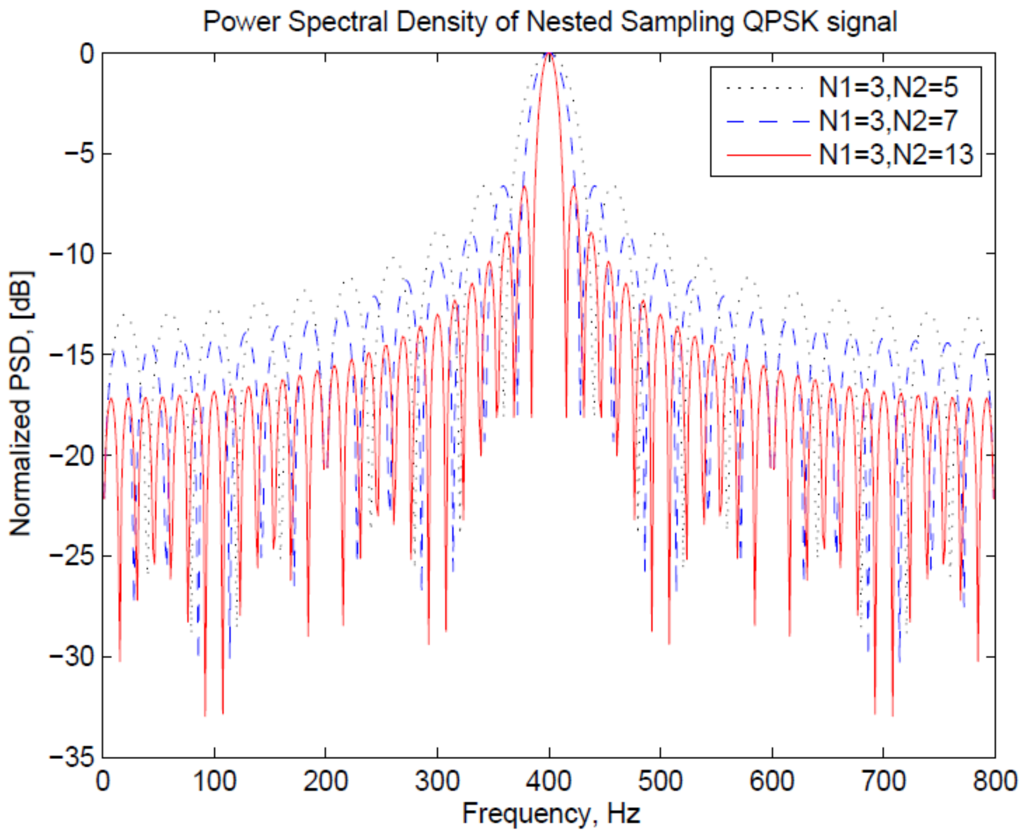


Fig. 17. PSD of Nested Sampling QPSK signal with different N2

Nested Sampling PSD (normalized)

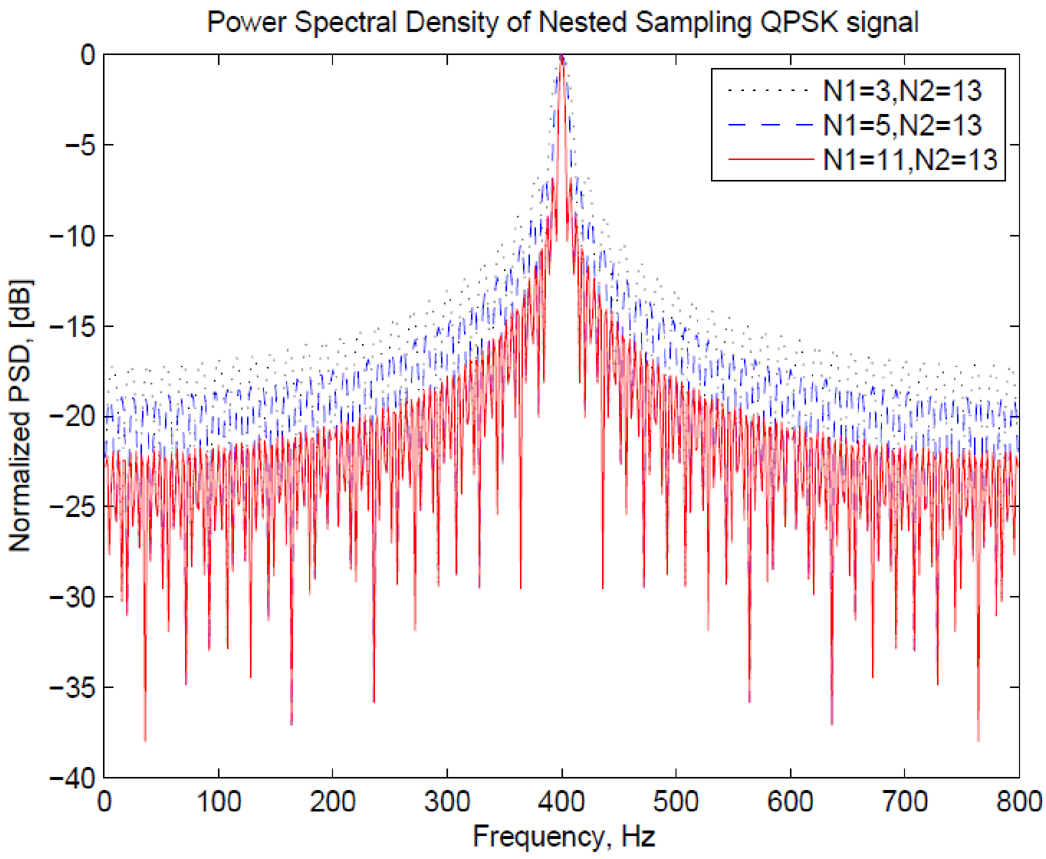
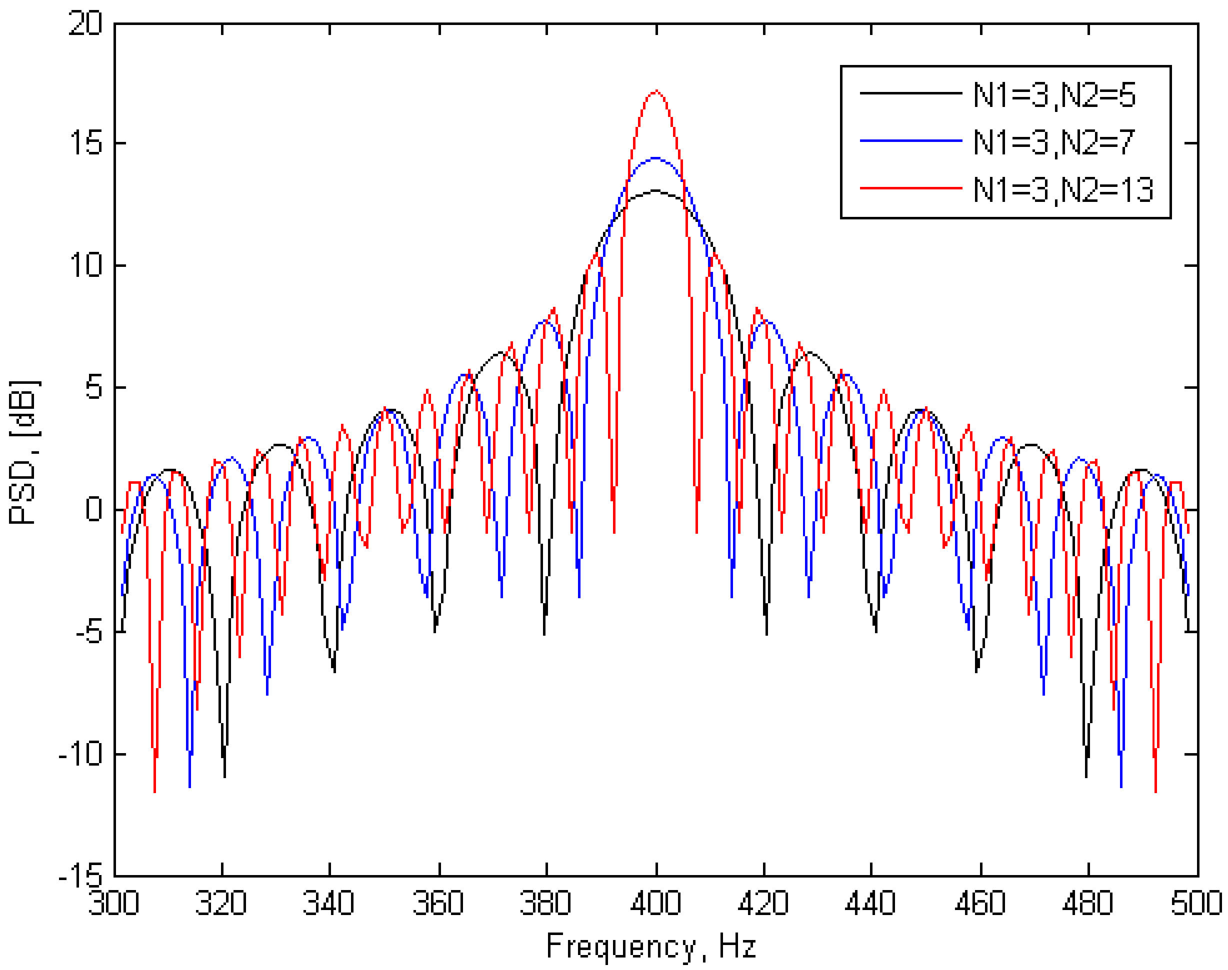


Fig. 18. PSD of Nested Sampling QPSK signal with different N1

Nested Sampling PSD (un-normalized)



Co-Prime Sampling PSD (normalized)

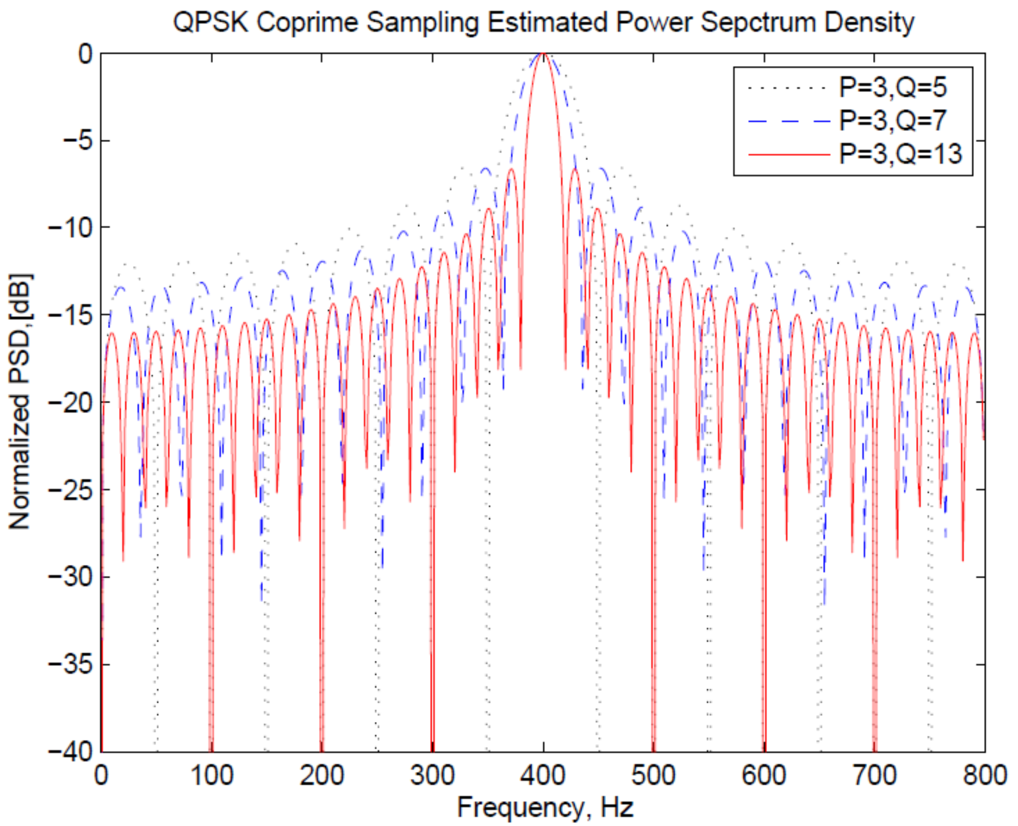


Fig. 19. PSD of Co-Prime Sampling QPSK signal with different Q

Co-prime Sampling PSD (normalized)

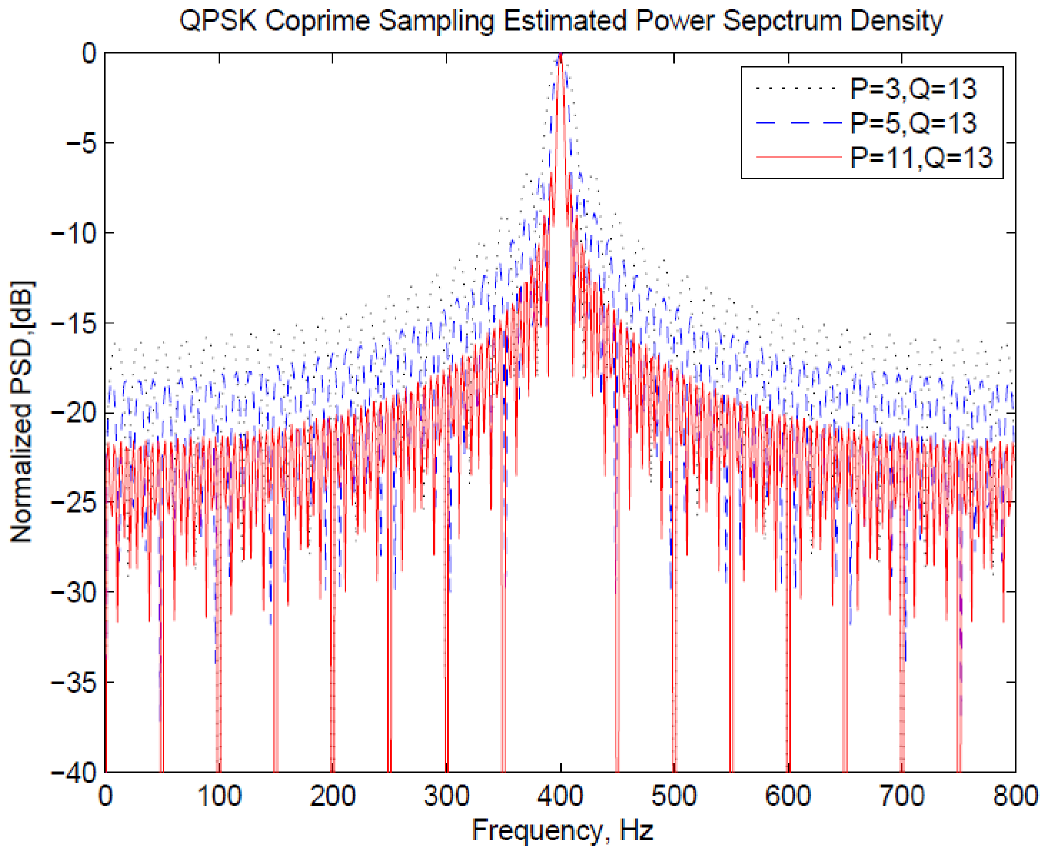
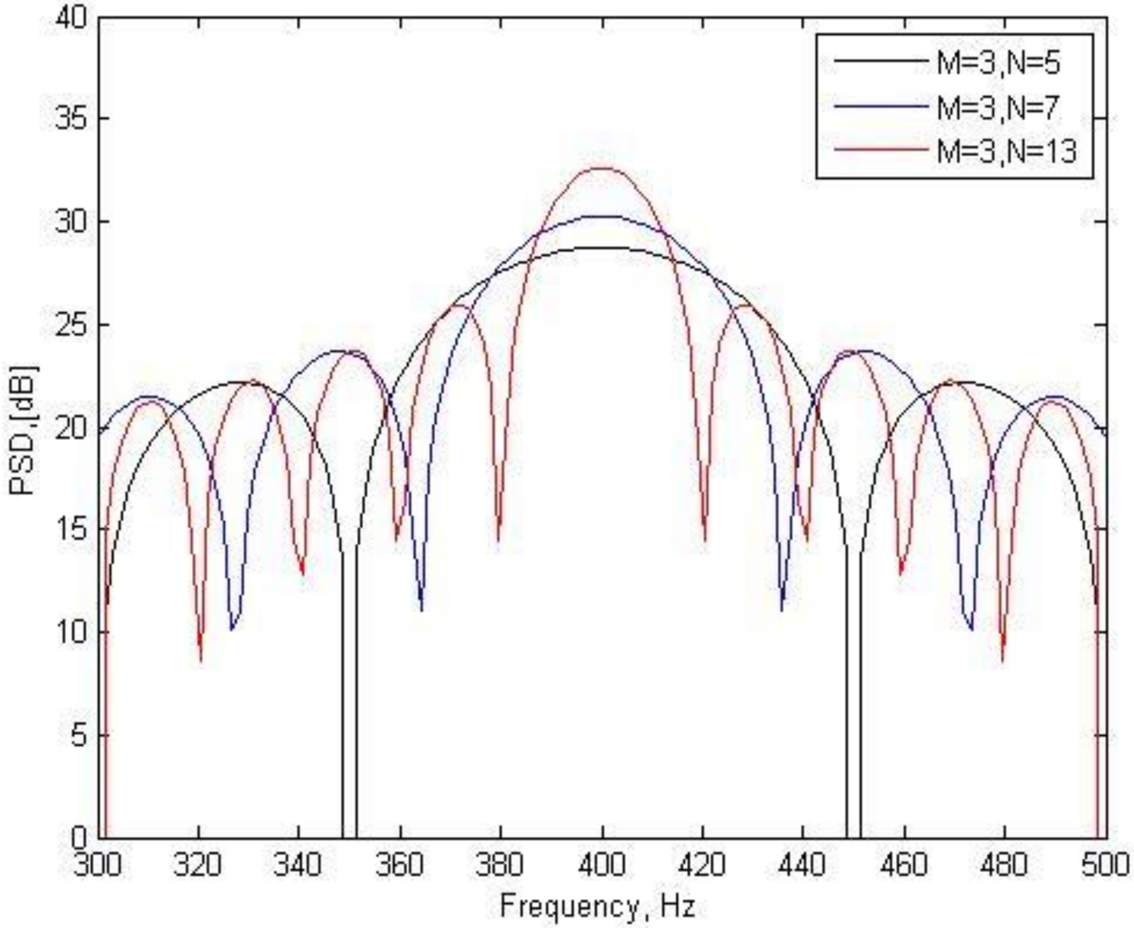
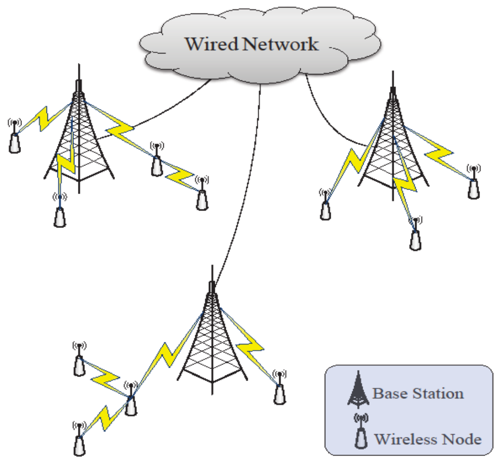


Fig. 20. PSD of Co-Prime Sampling QPSK signal with different P

Co-Prime Sampling PSD (un-normalized)



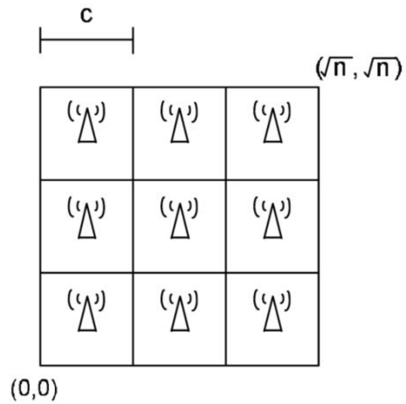
Application to Hybrid Wireless Network



- Wireless networks without support from the fixed infrastructure is known as ad-hoc networks.
- In other scenarios, a set of base stations are connected by wired links and placed within the ad-hoc networks to form a wired infrastructure. This resulting network is referred to as a hybrid wireless network.

Network Model

- Extended network in a square area
- *n* wireless nodes are placed randomly and uniformly



- *b* base stations partition the network into square cells of side length
- Base stations are linked together by wires with infinite bandwidth

Transmission Mode In Hybrid Wireless Network

- ## Intra-cell transmission mode

  When the source and destination are located in the same cell, the source transmits directly to the destination without the aid of the infrastructure.

- ## Infrastructure transmission mode

  When source and destination are located in different cells

  - **uplink phase** the source → base station
  - **transport phase** the data is transported via wired infra. network
  - **downlink phase** the base station → the destination

- **The total bandwidth of W Hz is partitioned into two orthogonal sub-channels with W_1 Hz and W_2 Hz , i.e., $W = W_1 + W_2$ (W_1 for nodes, W_2 for base stations).**

Outage Throughput Capacity over Slow Fading Channels

- If the signal duration is short compared to the channel coherent time, we have a slow fading channel.
- Outage capacity is the largest rate of transmission such that the outage probability P_{out} is less than ϵ .

Definition 1. For a hybrid wireless network of n nodes and b base stations, a ϵ-outage throughput capacity of $T_\epsilon(n, b)$ bit/s for each node is feasible if under the intra-cell mode or infrastructure mode, there is a spatial and temporal scheme for scheduling transmissions such that every node can *receive* $T_\epsilon(n, b)$ bit/s with the outage probability less than ϵ.

Outage Throughput Capacity under Intra-cell Mode

- **For wireless networks under intra-cell transmission mode, the outage capacity under Rayleigh fading**

$$C_{\epsilon}^{Ray} = \log\left\{1 + \epsilon^{\frac{1}{\kappa+1}} \cdot [(\kappa+1)!]^{\frac{1}{\kappa+1}} \cdot SNIR\right\}$$

where

$$SNIR = \frac{P \cdot min(1, \rho_{0d}^{-\alpha} e^{-\gamma \rho_{0d}})}{W_1 N_0 + I}.$$

κ is the number of transmitters in each cell

- For Spectrum Efficient waveform design, SNIR is much higher, so capacity is much higher with spectrum efficiency

Outage Throughput Capacity under Infrastructure Mode

- **For wireless networks under infrastructure mode, the downlink capacity under Rayleigh fading**

$$C_{\epsilon}^{Ray'} = \log\{1 + \epsilon \cdot SNIR'\}$$

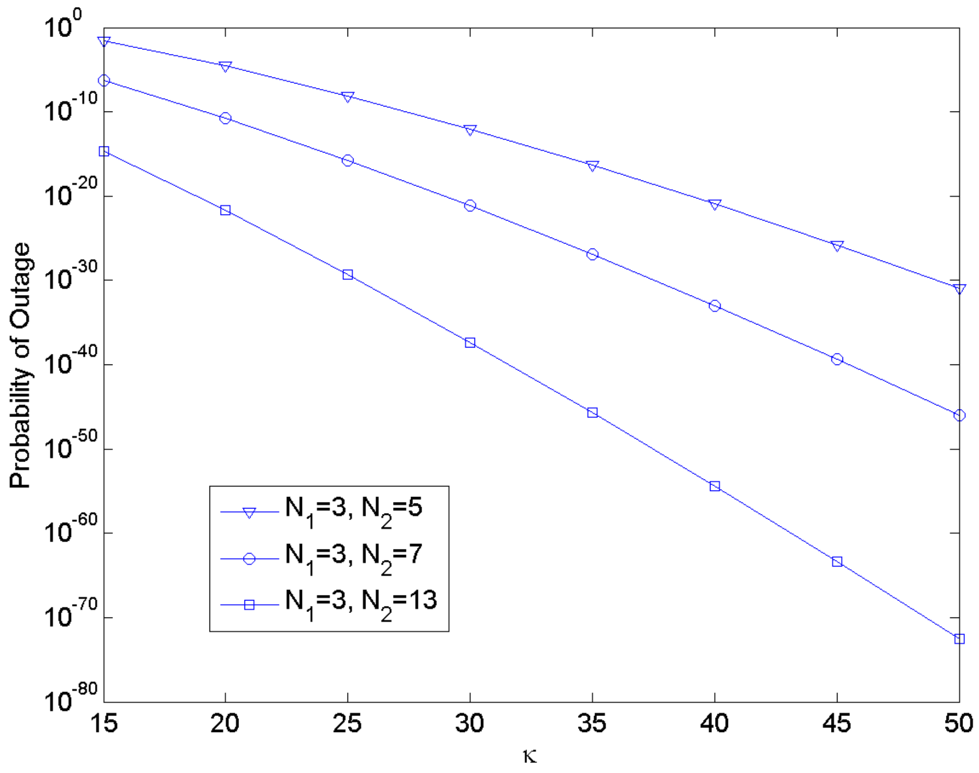
where

$$SNIR' = \frac{P \cdot min(1, \rho_{0d}^{-\alpha} e^{-\gamma \rho_{0d}})}{\lambda W_2 N_0 + I'}$$

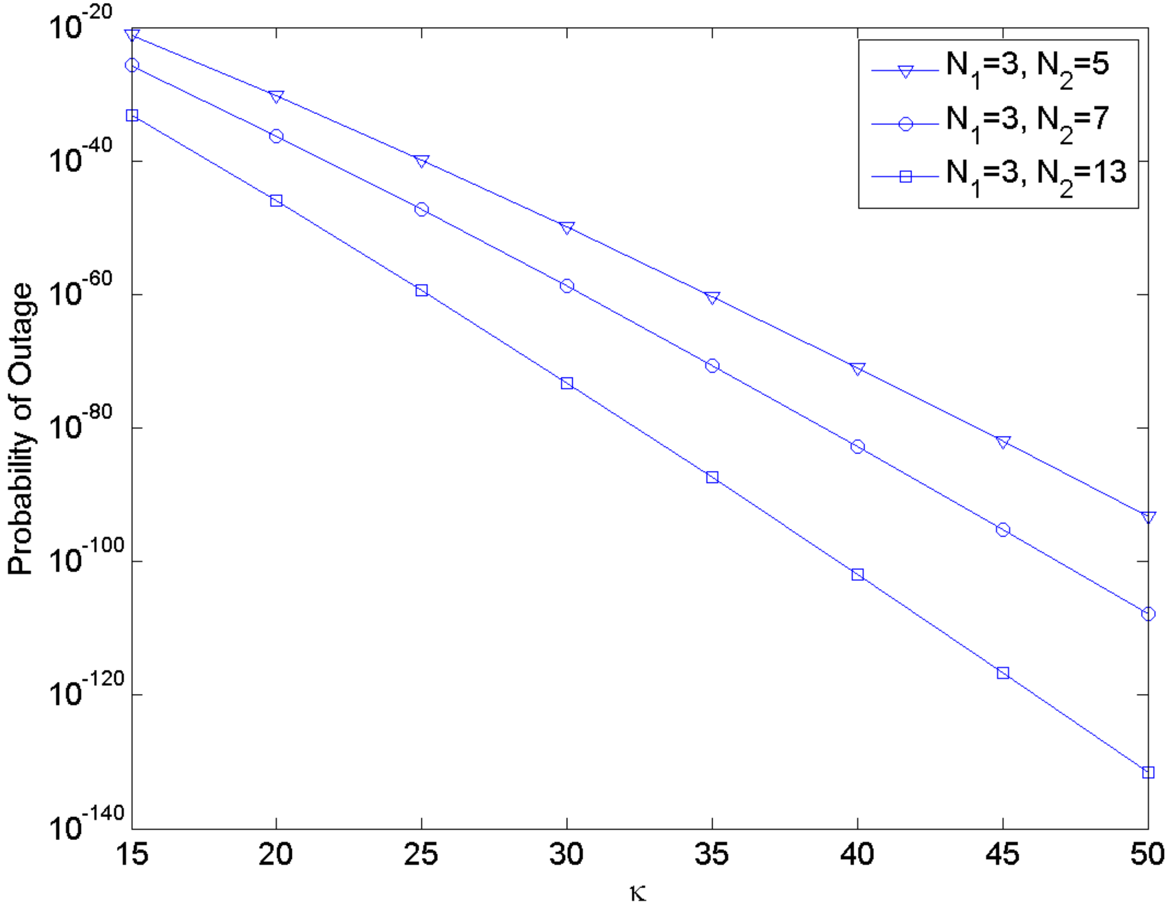
For Spectrum Efficient waveform design, $SNIR'$ is much higher

- Uplink Phase: Similar to inter-cell mode , the destination node is base station.
- Transport Phase: Wired infrastructure with infinite BW.
  No capacity limit.

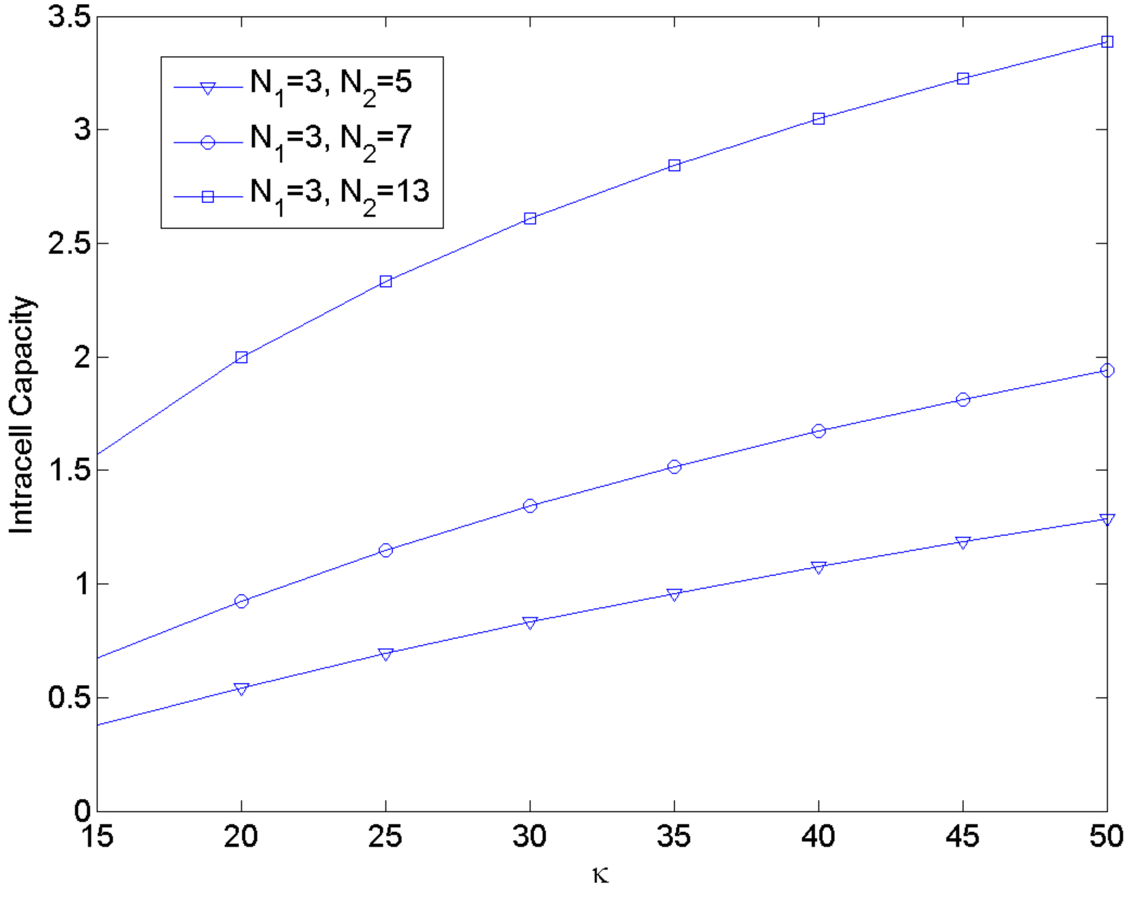
Outage Probabilities (Nested Sampling)



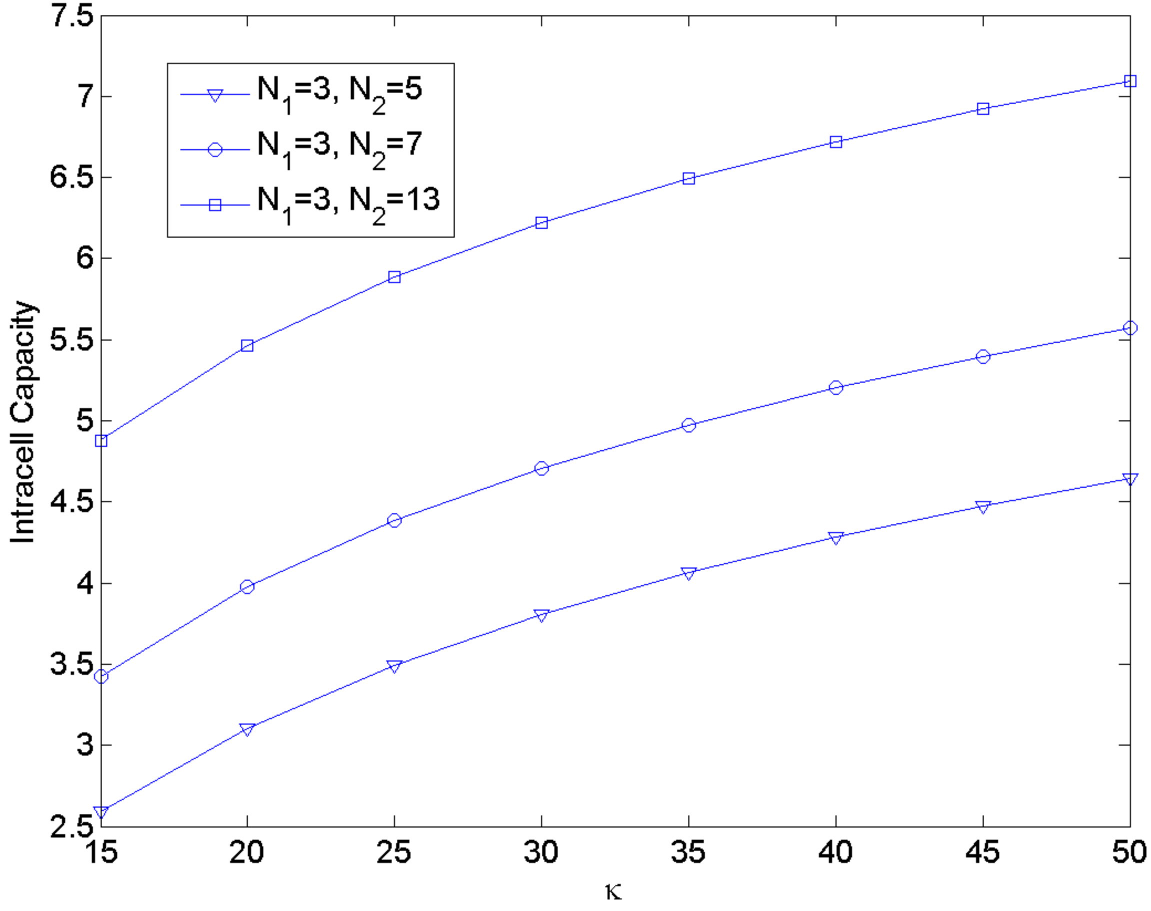
Outage Probabilities (Co-prime Sampling)



Capacities with outage 10^(-5) (Nested Sampling)



Capacities with outage 10^(-5) (Co-prime Sampling)



Conclusions

- Nested sampling and co-prime sampling are introduced
- Both sampling schemes could be used to generate spectrum efficient waveforms
- The spectrum efficient waveforms could be applied to hybrid wireless networks for lower outage probabilities and higher capacity.