

# Bayesian Rate Control Over Markovian Channel States

## Introduction

- The channel bandwidth: stochastic sequence,  $B(t)$
- The states  $i = 1, \dots, M$
- Transition matrix,  $P$
- Partially Observable Markov Decision Process
- Finite horizon  $T$ , time steps  $t = 1, \dots, T$
- Our belief vector at time  $t$ ,  $b_t = [b_t(1), \dots, b_t(M)]$

$$b_t(i) = \Pr(B(t) = i), i = 1, \dots, M$$

- Choose an action (rate)  $a$ ; if higher than  $B(t)$ , we will find out the exact value of  $B(t)$ , but incur a penalty.

- Evolution of belief vector :

$$b_{t+1} = \begin{cases} T_a b_t P, & \text{if } a \leq B(t) \\ I_{B(t)} P, & \text{if } a > B(t) \end{cases} \quad T_a b(i) = \begin{cases} 0, & \text{if } i < a \\ \frac{b(i)}{\sum_{j=a}^M b(j)}, & \text{if } i \geq a \end{cases}$$

- Immediate reward :

$$R(B(t); a) = \begin{cases} a, & \text{if } a \leq B(t) \\ B(t) - C(a - B(t)), & \text{if } a > B(t) \end{cases}$$

## Problem Formulation

- Policy vector:  $\pi = [\pi(1), \dots, \pi(T)]$
- Selecting an action  $\pi(t) = a_t \in \{1, \dots, M\}$
- Maximizing total discounted expected reward:

$$\max_{\pi} E^{\pi} \left[ \sum_{t=1}^T \beta^{t-1} R(b_t; a_t) \mid b_1 \right]$$

- defining value function, i.e. maximum expected remaining reward starting from time  $t$ :  $V_t(b)$

## Dynamic Programming

$$V_t(b_t) = \max_{a=1, \dots, M} V_t(b_t; a), \quad \forall t = 1, \dots, T$$

$$V_T(b_T; a) = \bar{R}(b_T; a),$$

$$V_t(b_t; a) = \bar{R}(b_t; a) + \beta V_t^f(b_t; a)$$

$$V_t^f(b_t; a) = E\{V_{t+1}(b_{t+1}) \mid a\}$$

$$= \sum_{i=a}^M b_t(i) V_{t+1}(T_a b_t P) + \sum_{i=1}^{a-1} b_t(i) V_{t+1}(I_i P)$$

## Optimal Policy

- Our goal is to find (at least structural properties of) the optimal policy.

$$a_t^{Optimal} = \arg \max_{a=1, \dots, M} V_t(b_t; a)$$

## Lower Bound Action

ignoring the impact of the current action on the future reward, myopic policy is given by

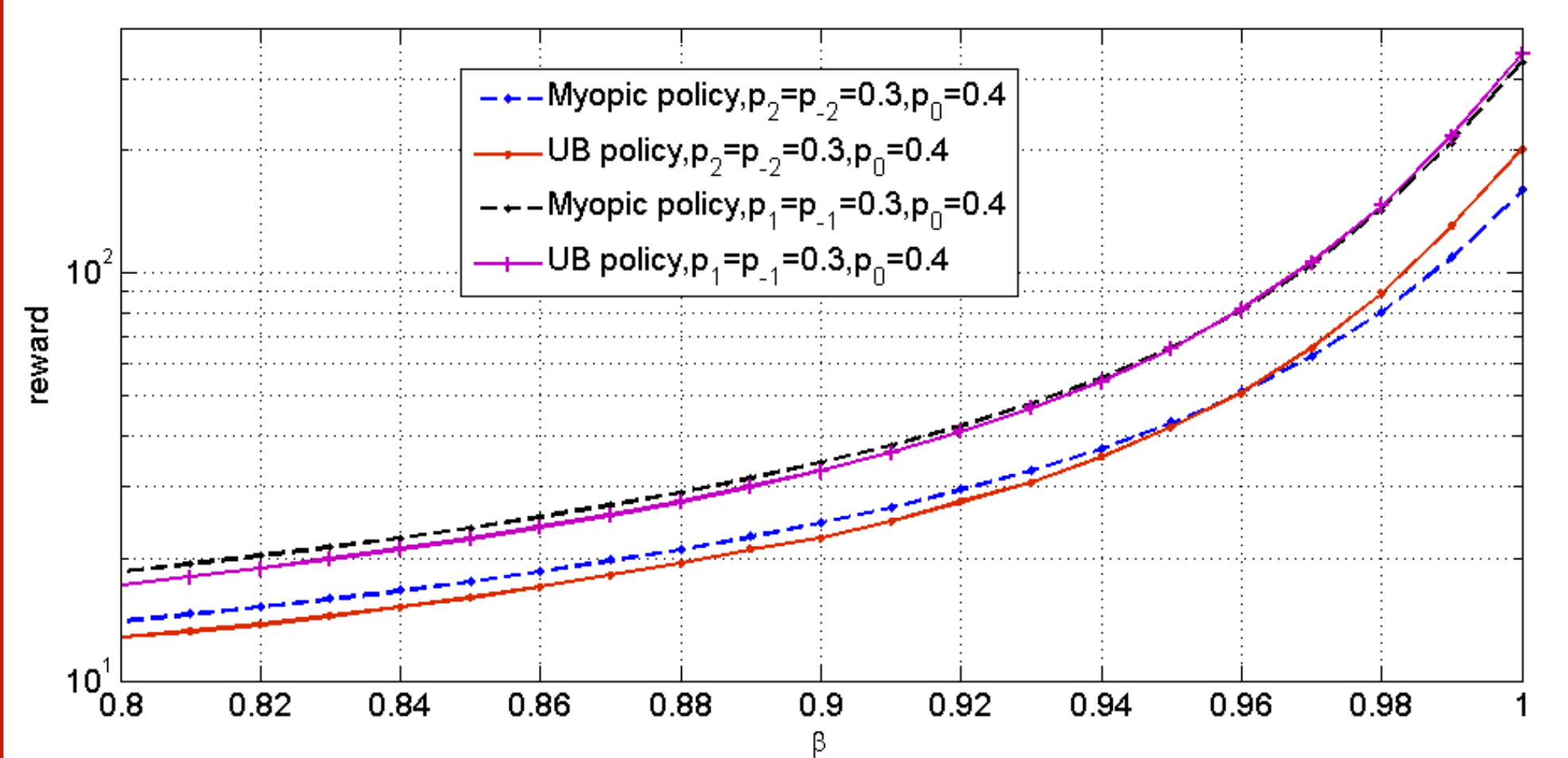
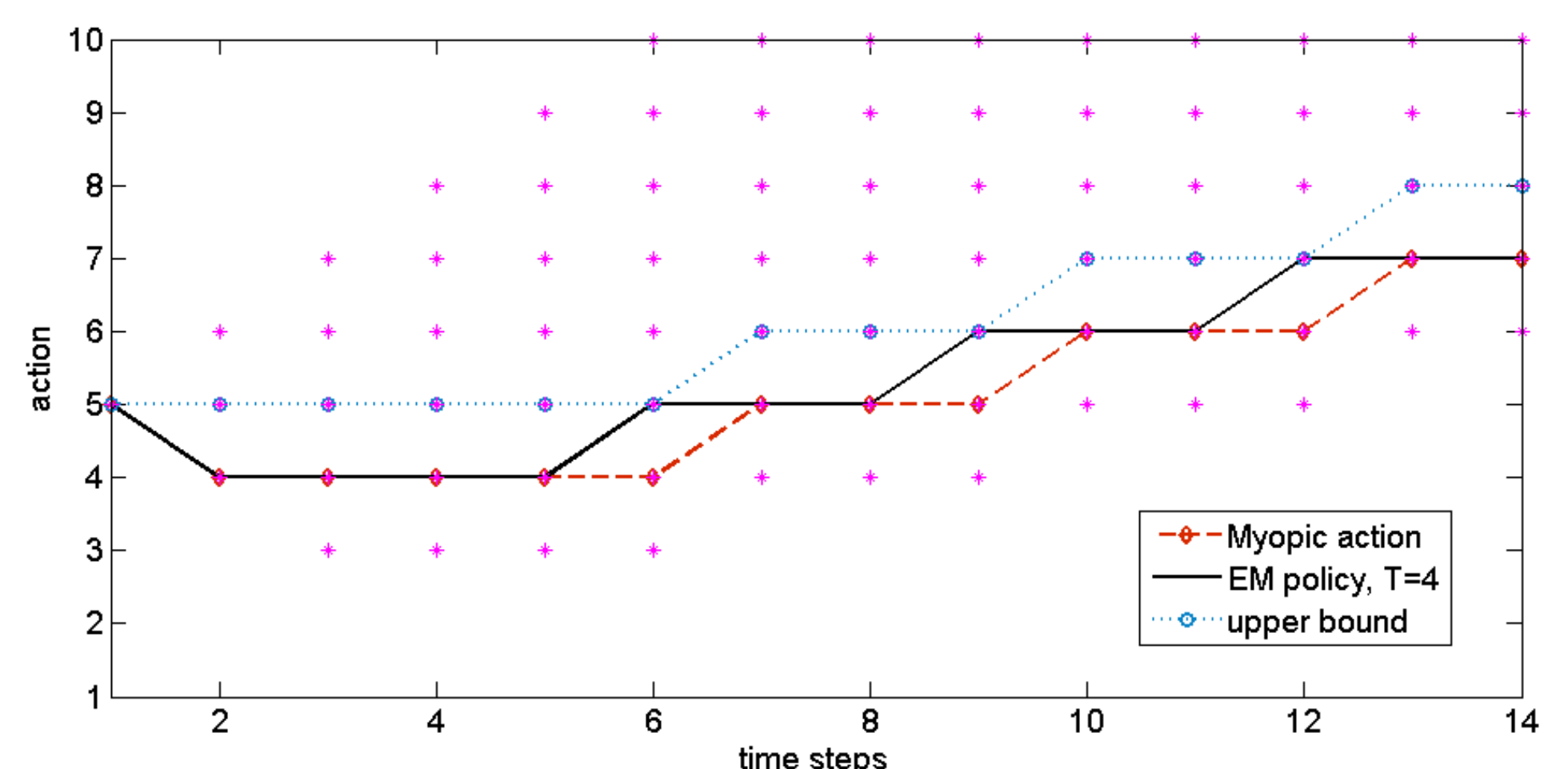
$$a_t^{Myopic}(b_t) = \arg \max_{a=1, \dots, M} \bar{R}(b_t; a) = \min \{a \mid \sum_{i=1}^a b_t(i) \geq \frac{1}{1+C}\}$$

## Upper Bound Action

$$a_t^{UB}(b_t) = \min \{a : \sum_{i=a+1}^M [\frac{\beta}{1-\beta}(i-a) + 1 + C] b_t(i) \leq C\}$$

- Assumption: The  $P$  matrix satisfies the State-Independent State Change (SISC) property.

## Simulation Results



Heuristic: Periodic Myopic-UB policy (some steps myopic and one step UB)

Length of period increases by  $C$  and decreases by  $\beta$

